Some Remarks on Antenna Response in a Reverberation Chamber

L. K. Warne and K. S. H. Lee

***Abstract*—The simple formula, $\langle P_r \rangle = (E_o^2/\eta)(\lambda^2/8\pi)$, for the received power of an antenna with a matched load in an over-moded cavity actually holds for an antenna of any shape and size. This can be seen from the close connection between the correlation tensor of the cavity field at two different points and the imaginary part of the free-space dyadic Green's function.**

***Index Terms*—Antenna response, correlation tensor, dyadic Green's function, reverberation chamber.**

I. Introduction

In a recent paper [1], Hill has shown that the statistical average of the received power of a linear dipole with a matched load and a *special* form of current distribution is given by

$$\langle P_r \rangle = \frac{E_o^2}{\eta}\,\frac{\lambda^2}{8\pi}. \tag{1}$$

First, we will show that (1) is valid for a linear antenna with any current distribution. To do this we notice that the correlation function $\rho_z(z_1, z_2)$ given by [1, eq. (18)] can be expressed as

$$\begin{aligned}\rho_z(z_1, z_2) &= \frac{3}{2k}\left(1 + \frac{1}{k^2}\,\frac{\partial^2}{\partial z_1^2}\right)\,\frac{\sin[k(z_1 - z_2)]}{z_1 - z_2}\\ &= \frac{6\pi}{k}\ \text{Im}\ \Gamma_{zz}\end{aligned} \tag{2}$$

where Im Γ_{zz} is the imaginary part of the zz-component of the free-space dyadic Green's function Γ [2]

$$\Gamma(\vec{r}_1, \vec{r}_2) = \frac{1}{4\pi}\left(\mathbf{u} + \frac{1}{k^2}\nabla_1\nabla_1\right)\frac{e^{ikr}}{r} \tag{3}$$

along the z-axis, with $r = |\vec{r}_1 - \vec{r}_2|$, $\mathbf{u}$ = unit dyad. One can show that (2) is indeed true by simply carrying out the differentiation, as is done in [2]. Now the power radiated, P_{rad}, by a linear antenna with current distribution $I(z)$ is given by

$$\begin{aligned}P_{\text{rad}} &= \frac{\omega\mu}{2}\iint \vec{J}(\vec{r}_1)\cdot \text{Im}\ \Gamma \cdot \vec{J}^*(\vec{r}_2)\ dV_1\ dV_2\\ &= \frac{\omega\mu}{2}\iint I(z_1)[\text{Im}\ \Gamma_{zz}(z_1 - z_2)]I^*(z_2)\ dz_1\ dz_2\\ &\equiv \frac{1}{2}R_d|I_o|^2\end{aligned} \tag{4}$$

Hence, the double integration in [1, eq. (22)] has the value

$$\iint \rho_z(z_1, z_2)I(z_1)I^*(z_2)\ dz_1\ dz_2 = \frac{3\lambda^2}{2\pi\eta}R_d|I_o|^2 \tag{5}$$

where we have made use of (2) and (4). Substitution of (5) in [1, eq. (22)] gives (1) for any current distribution on a linear antenna.

Next, we will show that formula (1) also holds for a matched antenna of any shape and size. To prove this we have to work out the statistical average of the correlation dyad of the electric field at two different points in the cavity, i.e., $\langle \vec{E}(\vec{r}_1)\vec{E}^*(\vec{r}_2)\rangle$. We define

$$R_{ij} = \frac{1}{2}[\langle E_i(\vec{r}_1)E_j^*(\vec{r}_2)\rangle + \langle E_i^*(\vec{r}_1)E_j(\vec{r}_2)\rangle], \qquad i, j = x, y, z. \tag{6}$$

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L. K. Warne is with Sandia National Laboratories, Albuquerque, NM 87185 USA.

K. S. H. Lee is with ITT Industries, Los Angeles, CA 90024 USA.

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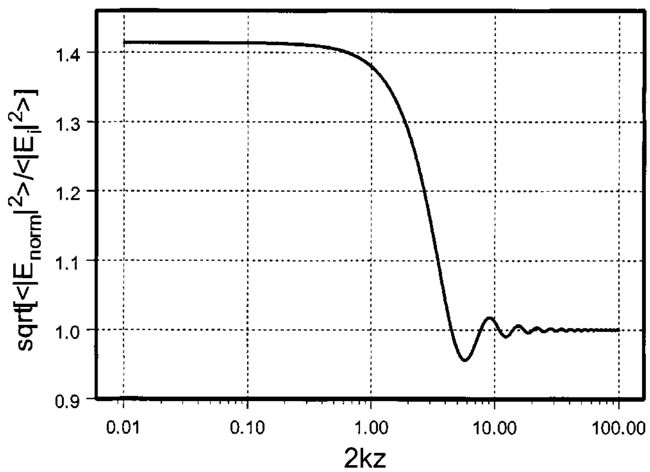

Fig. 1. The transition between the cavity wall region $z \to 0$ and the isotropic homogeneous region $z \to \infty$ is given by the formula in Dunn [5]. The ordinate is the square root of the mean square value of the electric field component normal to the wall divided by the mean square value of a single component in the isotropic homogeneous region. Note that $\langle |E_i|^2\rangle = E_o^2/3$, $i = x, y$, or z.

In view of (2) one should not be surprised to find that

$$R_{ij} = \frac{6\pi}{k}\ \text{Im}\ \Gamma_{ij} \tag{7}$$

To verify (7) we first carry out all the necessary differentiations in (3) and obtain

$$\begin{aligned}\frac{4\pi}{k}\ \text{Im}\ \Gamma_{ij} = &\left[3\frac{\sin u}{u^5} - 3\frac{\cos u}{u^4} - \frac{\sin u}{u^3}\right]u_i u_j\\ &+ \left[\frac{\sin u}{u} + \frac{\cos u}{u^2} - \frac{\sin u}{u^3}\right]\delta_{ij}\end{aligned} \tag{8}$$

where $u = kr$, $u_x = k(x_1 - x_2)$, $u_y = k(y_1 - y_2)$, etc., and δ_{ij} is the Kronecker delta. To work out the left-hand side of (7) we invoke what has been known in isotropic homogeneous turbulence, which enables one to write [3]

$$R_{ij} = -\frac{1}{2r}f' r_i r_j + (f + rf'/2)\delta_{ij} \tag{9}$$

where $r_x = x_1 - x_2$, etc. and $f' = df/dr$. The scalar function f can be identified with $\rho_z(z_1, z_2)$ in [1] and with $f_{||}(\xi)$ in [4], and is often referred to as the longitudinal correlation function given by

$$f(r) = \frac{3}{k^2r^2}\left(\frac{\sin kr}{kr} - \cos kr\right) = \frac{3}{u^2}\left(\frac{\sin u}{u} - \cos u\right). \tag{10}$$

Using (10) in (9) gives 3/2 times the right-hand side of (8), and thus (7) is verified. Hence,

$$\begin{aligned}\langle P_r \rangle &= \frac{E_o^2}{12R_d|I_o|^2}\iint J_i(\vec{r}_1)R_{ij}(r)J_j^*(\vec{r}_2)\ dV_1\ dV_2\\ &= \frac{E_o^2}{\eta}\,\frac{\lambda^2}{8\pi}\end{aligned} \tag{11}$$

where repeated indices are summed. Note that the trace of R_{ij} is

$$R_{xx} + R_{yy} + R_{zz} = 3\frac{\sin kr}{kr}. \tag{12}$$

Thus far, we have proved that (1) is valid for a matched receiving antenna of any shape and size. The underlying assumption for (1) to hold is that the cavity is highly over-moded in the sense that many overlapping modes are present within the bandwidth of one resonant mode. It would be interesting to show how the simple (1) would be modified for a receiving antenna in an under-moded cavity where the frequency is still quite high but operates in a range where the spectra of the resonant modes are well separated.

Before concluding two remarks are in order. It is worthwhile to mention that formula (1) can also be obtained by averaging the effective receiving area A_e over all angles of incidence and polarization of an incident plane wave in free space, i.e.,

$$\langle P_r \rangle = \frac{E_o^2}{\eta} \langle A_e \rangle = \frac{E_o^2}{\eta} \left\langle \frac{\lambda^2}{4\pi} Gpq \right\rangle = \frac{E_o^2}{\eta} \frac{\lambda^2}{8\pi} \tag{13}$$

since $\int G \, d\Omega/4\pi = 1$, $\int p \, d\chi/2\pi = 1/2$ and $q = 1$ for a matched load, where G is the antenna's directivity gain and p the polarization mismatch factor [2]. The other point to be made is that there is a boundary layer near the cavity wall, which is about one wavelength thick (see Fig. 1 and [5]). Fig. 1 illustrates the enhancement of the mean square normal field as the wall is approached. It is not clear if formula (1) needs to be changed when part of the receiving antenna is immersed in the layer.

Although the result (13) may be familiar to those working with mode-stirred chambers [6]–[8], the rigorous derivation of it using the correlation tensor of the cavity fields is new. What is also new is the result (7) that connects the correlation tensor and the imaginary part of the free-space dyadic Green's function.

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Simple Estimation of Equivalent Magnetic Dipole Moment to Characterize ELF Magnetic Fields Generated by Electric Appliances Incorporating Harmonics

Kenichi Yamazaki and Tadashi Kawamoto

***Abstract*—A simple method of quantifying the ELF (extremely low frequency) magnetic field distribution around electric appliances, which takes harmonics into account, is newly proposed. The proposed method involves: 1) a simple estimation of the position of an equivalent magnetic dipole moment inside an appliance, using two magnetic field meters; 2) identification of the amplitude of the dipole moment by magnetic-field measurements at certain points; and 3) calculation of the magnetic field distribution around the appliance using the estimated dipole moment. In this method, the dipole moment vector is assumed to be a scalar value by allowing an uncertainty of 6dB in the estimated magnetic field, which enables easy estimation of the dipole moment. In addition, frequency characteristics of the magnetic field are taken into account by considering the harmonic components in the magnetic field waveform. The proposed method was applied to 13 types of appliances, and their equivalent magnetic dipole moments and harmonic components were determined. The results revealed that the proposed method is applicable to many electric appliances. The conditions required for the adoption of the method were also clarified.**

***Index Terms*—Electric appliance, ELF magnetic field, harmonics, magnetic dipole moment.**

I. Introduction

Quantitative understanding of magnetic field characteristics in certain circumstances still remains a major issue regarding the biological effects of ELF (extremely low frequency) magnetic fields. Power lines and electric appliances are considered major sources of ELF magnetic fields in occupational or residential environments. The power line magnetic field has been well characterized by means of a current dipole moment introduced by Kaune and Zaffanella [1], where the magnetic field distribution around an infinite length of a straight line conductor is simply characterized by the combination of several orders of the moments and certain orders of inverse power of the distance between the position of the conductor and the observation point. On the other hand, magnetic fields generated by electric appliances are not well characterized because of their diversities in features such as shapes, sizes, and driving circuits. In addition, recently, the frequency components of the magnetic field higher than the power frequency and the rapid change in amplitude of the field have been focused on as a "transient magnetic field" because the higher frequency magnetic field induces higher current inside living bodies [2]. In the discussion, electric appliances are considered one of the major sources of the transient magnetic field because they are generally rich in harmonics [3].

Considering these backgrounds, an appropriate method of quantifying the characteristics of magnetic fields generated by electric appliances is desirable. The aim of our study is to propose a simple method of characterizing the ELF magnetic field from appliance, incorporating harmonic components.

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The authors are with the Electrical Insulation Deptartment, Central Research Institute of Electric Power Industry (CRIEPI), Komae-shi, Tokyo 201-8511, Japan.

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